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Opposition Inferences and Generalized Quantifiers

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Abstract

Opposition inferences constitute an important type of immediate inferences studied in Classical Logic. These are inferences involving 4 types of relations defined on the classical square of opposition: subalternation, contradictoriness, contrariety and subcontrariety. Based on the definitions of these relations, one can immediately obtain certain opposition inferences, such as the following:

(1)   (Given that there is some student.)

No student sang. ╞ It is not the case that every student sang.

However, the applicability of classical opposition inferences is limited because Classical Logic only studied quantified statements headed by the 4 classical quantifiers: "every", "no", "some" and "not every". The advent of modern Generalized Quantifier Theory (GQT) has opened up possible ways to extend the classical opposition inferences. Not only can we consider opposition inferences of quantified statements headed by non-classical quantifiers such as "most", "at least 3/4", but we can now consider inferences that have very different structures than those above, such as the following:

(2)   (Given that there is some student.)

Every member is elderly. ╞ It is not the case that every member is a teenager.

Note that although (1) and (2) both make use of the contrary relation, the contrariety in (1) is between the quantifiers "no" and "every", whereas the contrariety in (2) is between the predicates "be elderly" and "be a teenager".

In this paper I will develop a new theory on opposition inferences using some notions and results of modern GQT. Having defined the basic notions associated with opposition inferences, I will then propose and prove a number of theorems concerning the opposition inference patterns of various types of quantifiers. These theorems will enable us to determine and explain the validity of opposition inferences such as the following:

(3)   At least 3/4 of the members are teenagers. ╞ It is not the case that at least 3/4 of the members are elderly.

(4)   Less than 1/4 of the members are aged over 50. ╞ It is not the case that less than 1/4 of the members are aged below 51.

Note that while the predicates "be teenagers" and "be elderly" in (3) satisfy the

contrariety relation, the predicates "be aged over 50" and "be aged below 51" in (4) satisfy the subcontrariety relation. The different inference patterns can be explained by the different logical properties of the quantifiers "at least 3/4" and "less than 1/4".

As modern GQT has also studied various types of complex quantifiers, such as "all except X", "the same … as …", we can also discover the opposition inference patterns of these quantifiers. More importantly, I will propose a principle that enables us to discover the opposition inference patterns of sentences with multiple quantifiers based on the inference patterns of the individual quantifiers, such as the following:

(5) (Given that every company employed somebody and some company went bankrupt.)

No company employing somebody aged over 50 went bankrupt. ╞ It is not the case that no company employing somebody aged below 51 went bankrupt.

Using the principle proposed in this paper, one can easily check the validity of the above inference, which is otherwise not a straightforward matter.

It will turn out that the notions of opposition inferences developed in this paper are analogous to those of monotonicity inferences studied under modern GQT. The present study can thus be seen as a generalization and extension of the studies on monotonicity inferences.