Internal Reading and Reciprocity*

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Abstract. This paper first points out that the internal reading of sentences containing certain reciprocal items such as "the same" carries a reciprocal meaning. It then provides a uniform formal treatment for these items that reflects the reciprocal meaning by using the Generalized Quantifier Theory enriched by the notions of generalized noun phrases (GNPs) and raised verb phrases. The basic framework initially makes explicit use of the GNP EACH OTHER, and is then refined to make EACH OTHER implicit so as to be in line with the surface syntactic structure. The framework is further refined to handle cases that are different from the basic cases, including reciprocal items appearing in non-object positions, conjoined phrases containing reciprocal items, downward monotonic and non-monotonic quantifiers, weaker reciprocal meaning and non-NP triggers.

Keywords: internal reading, reciprocal items, Generalized Quantifier Theory, generalized noun phrases, raised verb phrases, polymorphism

1 Introduction

Sentences with "each other" in English have been treated as the canonical reciprocal expressions. However, apart from "each other", some other sentences also carry a reciprocal meaning. A typical example is the following sentence:

$$\text{John, Peter and Mary read the same books.} \tag{1}$$

The phrase with "the same" above is ambiguous with two readings: the deictic reading and the internal reading. Under the deictic reading, "same" means "same as something mentioned in the previous discourse or salient in the context",

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whereas under the internal reading, the sentence above is equivalent to the following (as pointed out by [12]):

John, Peter and Mary read the same books as each other. (2)

meaning that any two of the three persons read the same books (here I assume the strongest reciprocal meaning as discussed in [5] and [11] by default). In this paper, I will not discuss the independence between these two readings and will focus on the internal reading. Note that the internal reading of sentence (1) is triggered by the plural noun phrase (NP) "John, Peter and Mary". If it is replaced by a singular NP such as "John", then the internal reading of the sentence disappears. In Subsection 4.5, I will discuss other internal reading triggers.

In addition to "the same", some other lexical items, hereinafter called "reciprocal items" and including "different", "opposite", "similar", "related", "separate", "adjacent", "complementary", "connected", "disjoint", etc, behave similarly in that a sentence containing one of these items has an internal reading carrying a reciprocal meaning. This is best illustrated by the following example which sounds weird:

*Particles 1, 2 and 3 have different charges. (internal reading) (3)

Under the intended (namely internal) reading, the sentence above means that the charges of the three particles are different from each other, i.e. particles 1 and 2 have different charges, particles 2 and 3 have different charges, and particles 3 and 1 have different charges. But this is impossible given that there are only two possible charges (positive vs negative).

Another evidence for the reciprocal meaning of these items is that many of them can be rendered in Chinese as words containing the morpheme "xiang" or "hu" meaning "mutual" as shown in the following table which gives the romanization and Chinese characters of the Chinese renditions of the reciprocal items, although these may not be the most colloquial Chinese renditions for these items.

Note that "xiang" and "hu" can in fact appear in Chinese reciprocal expressions. For instance, "xiang ai", where "ai" is the Chinese word for "love", means "love each other". The two morphemes can even combine to make the compound adverb "huxiang" or "xianghu" meaning "mutually". For instance, "huxiang bangzhu", where "bangzhu" is the Chinese word for "help", means "help each other".

In this paper, I will provide a uniform formal treatment for the internal reading of all the reciprocal items that reflects the reciprocal meaning. But first I have to point out that when I say a sentence containing a reciprocal item other than "the same" has an internal reading, I mean that the sentence carries a reciprocal meaning just like (2), and do not mean that the sentence is necessarily ambiguous in that it also has a deictic reading. In fact, sentences containing many of these reciprocal items (other than "the same") are not ambiguous. For example, the following sentence does not have a deictic reading:

Reciprocal Item	Chinese Rendition (Romanization)	Chinese Rendition (Chinese Characters)
the same	*xiangtong*	相同
different	*xiangyi*	相異
opposite	*xiangfan*	相反
similar	*xiangsi*	相似
related	*xiangguan*	相關
separate	*xiangge*	相隔
adjacent	*xianglin*	相鄰
complementary	*hubu*	互補
connected	*hutong*	互通
disjoint	*huchi*	互斥

John, Peter and Mary live on separate floors. (4)

2 Formal Preliminaries

In this paper, I will adopt the notation in [15] which uses small cap font and boldface font for the denotations of logical and non-logical terms, respectively. If f is a characteristic function, then f' is the set of entities that f characterizes. The formal framework is mainly based on the Generalized Quantifier Theory (GQT) enriched by the notions of generalized noun phrases (GNPs) and raised verb phrases (raised VPs) developed by [18], [19], [21], [22], etc.

Before introducing the notion of GNP, I first review some basic notions of GQT. Under the classical GQT, a generalized quantifier (GQ) is a function with a number of predicates as arguments and a truth value as output. The arities of the arguments are used to name the type of a GQ. For example, a proper name such as "John" is treated as a type $\langle 1 \rangle$ GQ $I_{\mathbf{j}}$ (called "Montagovian individual" in the GQT literature) corresponding to the individual term **j** (short for "John") which requires a unary predicate as its sole argument and outputs a truth value with the following definition[1]:

$$I_{\mathbf{j}} = \lambda P_{et}[P(\mathbf{j}) = 1] \quad (5)$$

A determiner such as "fewer than 3" is treated as a type $\langle 1, 1 \rangle$ GQ FEWER THAN 3 which requires two unary predicates as its arguments and outputs a truth value.

[1] In what follows, a subscript attached to a variable (such as *et* attached to P in (5)) is the type of that variable.

In this paper, I will also make use of the case extension operators proposed by [8]. Let Q be a type $\langle 1 \rangle$ GQ, then the nominative and accusative extensions of Q, notated Q_{nom} and Q_{acc} respectively, are functions defined as follows:

$$Q_{\text{nom}} = \lambda R_{e^2t} \lambda w_e [Q(\lambda z_e [R(z, w) = 1]) = 1] \tag{6}$$

$$Q_{\text{acc}} = \lambda R_{e^2t} \lambda z_e [Q(\lambda w_e [R(z, w) = 1]) = 1] \tag{7}$$

Applying any one of these two functions to a binary predicate is tantamount to filling one of the arguments (nominative or accusative) of the binary predicate with the NP represented by Q, thus obtaining a unary predicate. For example, by applying $(I_{\mathbf{j}})_{\text{nom}}$ and $(I_{\mathbf{j}})_{\text{acc}}$ to the binary predicate **hit**, we obtain the unary predicates $(I_{\mathbf{j}})_{\text{nom}}(\mathbf{hit})$ and $(I_{\mathbf{j}})_{\text{acc}}(\mathbf{hit})$, meaning "that which John hit" and "that which hit John", respectively.

Very often, we need to express conjoined phrases such as "John, Peter and Mary". In this paper, I use a polymorphic Boolean operator $\text{AND}_{n,\tau}$ to conjoin n expressions of type τ with the following recursive definition adapted from [14]:

$$\text{AND}_{n,\tau} = \begin{cases} \lambda((p_1)_t, \ldots, (p_n)_t)[p_1 \wedge \ldots \wedge p_n] & \text{if } \tau = t \\ \lambda((X_1)_\tau, \ldots, (X_n)_\tau)\lambda Y_{\sigma_1}[\text{AND}_{n,\sigma_2}(X_1(Y), \ldots, X_n(Y))] & \text{if } \tau = \sigma_1\sigma_2 \end{cases} \tag{8}$$

Thus, "John, Peter and Mary" can be denoted as $\text{AND}_{3,(et)t}(I_{\mathbf{j}}, I_{\mathbf{p}}, I_{\mathbf{m}})$[2], where $I_{\mathbf{p}}$ and $I_{\mathbf{m}}$ are the Montagovian individuals corresponding to the individual terms **p** (short for "Peter") and **m** (short for "Mary"), respectively. It can be shown that

$$\text{AND}_{3,(et)t}(I_{\mathbf{j}}, I_{\mathbf{p}}, I_{\mathbf{m}}) = \lambda P_{et}[P(\mathbf{j}) = 1 \wedge P(\mathbf{p}) = 1 \wedge P(\mathbf{m}) = 1] \tag{9}$$

From this we obtain

$$\text{AND}_{3,(et)t}(I_{\mathbf{j}}, I_{\mathbf{p}}, I_{\mathbf{m}})' = \{S : \{\mathbf{j}, \mathbf{p}, \mathbf{m}\} \subseteq S\} \tag{10}$$

There are three useful notions associated with type $\langle 1 \rangle$ GQs in a finite universe, namely live-on sets, topic sets and witness sets. The definitions of these notions according to [13] are given as follows. Let Q be a type $\langle 1 \rangle$ GQ in a finite universe (which is a reasonable assumption for linguistic applications), then a set of individuals S is a live-on set of Q if the following is true for any set of individuals T:

$$T \in Q' \text{ iff } T \cap S \in Q' \tag{11}$$

The topic set of Q is the smallest live-on set of Q. A set of individuals W is a witness set of Q if W is a subset of the topic set of Q and $W \in Q'$. In what follows, I will use $\text{Li}(Q)$ to denote the set of live-on sets of Q, $\text{Top}(Q)$ to denote

[2] Note that under the e-t notation, a type $\langle 1 \rangle$ GQ is a function of type $(et)t$.

the (unique) topic set of Q, and Wit(Q) to denote the set of witness sets of Q. Note that while Top(Q) is a set of individuals, Li(Q) and Wit(Q) are sets of sets of individuals.

For example, we have $\mathrm{Li}(\textsc{and}_{3,(et)t}(I_{\mathbf{j}}, I_{\mathbf{p}}, I_{\mathbf{m}})) = \textsc{and}_{3,(et)t}(I_{\mathbf{j}}, I_{\mathbf{p}}, I_{\mathbf{m}})'$. To prove this, we first take an arbitrary member S of $\textsc{and}_{3,(et)t}(I_{\mathbf{j}}, I_{\mathbf{p}}, I_{\mathbf{m}})'$, which according to (10) is a set S such that $\{\mathbf{j}, \mathbf{p}, \mathbf{m}\} \subseteq S$, and show that S satisfies (11) for any set of individuals T where Q is instantiated as $\textsc{and}_{3,(et)t}(I_{\mathbf{j}}, I_{\mathbf{p}}, I_{\mathbf{m}})$. Given that $\{\mathbf{j}, \mathbf{p}, \mathbf{m}\} \subseteq S$, we have $\{\mathbf{j}, \mathbf{p}, \mathbf{m}\} \subseteq T$ iff $\{\mathbf{j}, \mathbf{p}, \mathbf{m}\} \subseteq T \cap S$. According to (10), we thus have $T \in \textsc{and}_{3,(et)t}(I_{\mathbf{j}}, I_{\mathbf{p}}, I_{\mathbf{m}})'$ iff $T \cap S \in \textsc{and}_{3,(et)t}(I_{\mathbf{j}}, I_{\mathbf{p}}, I_{\mathbf{m}})'$.

We next take an arbitrary set S which is not a member of $\textsc{and}_{3,(et)t}(I_{\mathbf{j}}, I_{\mathbf{p}}, I_{\mathbf{m}})'$ and show that S does not satisfy (11) for a particular set $T = \{\mathbf{j}, \mathbf{p}, \mathbf{m}\}$. Without loss of generality, we may assume that $\mathbf{j} \notin S$. It then follows that $\mathbf{j} \notin T \cap S$ and so $\{\mathbf{j}, \mathbf{p}, \mathbf{m}\} \not\subseteq T \cap S$. According to (10), we thus have $T \in \textsc{and}_{3,(et)t}(I_{\mathbf{j}}, I_{\mathbf{p}}, I_{\mathbf{m}})'$ but $T \cap S \notin \textsc{and}_{3,(et)t}(I_{\mathbf{j}}, I_{\mathbf{p}}, I_{\mathbf{m}})'$.

Since $\mathrm{Top}(\textsc{and}_{3,(et)t}(I_{\mathbf{j}}, I_{\mathbf{p}}, I_{\mathbf{m}}))$ is the smallest member of $\mathrm{Li}(\textsc{and}_{3,(et)t}(I_{\mathbf{j}}, I_{\mathbf{p}}, I_{\mathbf{m}}))$, we have $\mathrm{Top}(\textsc{and}_{3,(et)t}(I_{\mathbf{j}}, I_{\mathbf{p}}, I_{\mathbf{m}})) = \{\mathbf{j}, \mathbf{p}, \mathbf{m}\}$. Moreover, since $\{\mathbf{j}, \mathbf{p}, \mathbf{m}\}$ is the only set satisfying $\{\mathbf{j}, \mathbf{p}, \mathbf{m}\} \subseteq \mathrm{Top}(\textsc{and}_{3,(et)t}(I_{\mathbf{j}}, I_{\mathbf{p}}, I_{\mathbf{m}}))$ and $\{\mathbf{j}, \mathbf{p}, \mathbf{m}\} \in \textsc{and}_{3,(et)t}(I_{\mathbf{j}}, I_{\mathbf{p}}, I_{\mathbf{m}})'$, we have

$$\mathrm{Wit}(\textsc{and}_{3,(et)t}(I_{\mathbf{j}}, I_{\mathbf{p}}, I_{\mathbf{m}})) = \{\{\mathbf{j}, \mathbf{p}, \mathbf{m}\}\} \tag{12}$$

The notion of GNP is a generalization of GQ. Instead of outputting truth values, a GNP outputs a characteristic function of ordered n-tuples of individuals (represented by ": n") or a characteristic function of type $\langle 1 \rangle$ GQs (represented by ": $\langle 1 \rangle$"). Following [19], I will treat "each other" as a GNP with a binary predicate as input and a set of type $\langle 1 \rangle$ GQs as output, and its type is notated $\langle 2 : \langle 1 \rangle \rangle$. As for the denotation of this GNP, I will adopt a modified version of the denotation of the most basic reciprocal quantifier in [10] which is given as follows:

$$\textsc{each other} = \lambda R_{e^2t} \lambda Q_{(et)t} [\exists X \in \mathrm{Wit}(Q)[|X| \geq 2 \wedge X^2 - Id_X^2 \subseteq R']] \tag{13}$$

In the above, Q is a variable of upward monotonic type $\langle 1 \rangle$ GQs. Id_X^2 is the set $\{(x, x) : x \in X\}$ and so the statement $X^2 - Id_X^2 \subseteq R'$ means that every member x of X stands in the relation R with every other member of X except perhaps x itself. For instance, the following sentence

John, Peter and Mary hit each other. (14)

will be denoted by

$$\textsc{each other}(\mathbf{hit})(\textsc{and}_{3,(et)t}(I_{\mathbf{j}}, I_{\mathbf{p}}, I_{\mathbf{m}})) \tag{15}$$

According to (13), the expression above is true if and only if there exists a plural witness set of $\textsc{and}_{3,(et)t}(I_{\mathbf{j}}, I_{\mathbf{p}}, I_{\mathbf{m}})$ (which, as discussed above, must be the set

$\{\mathbf{j}, \mathbf{p}, \mathbf{m}\}$) such that every member of that set hits every other member of that set, which is exactly what (14) means.

A consequence of the aforesaid treatment is that EACH OTHER(**hit**) denoting the VP "hit each other" has type $((et)t)t$ instead of et. Thus, "hit each other" is an example of raised VPs studied in [22]. The main difference between an ordinary VP (of type et) and a raised VP (of type $((et)t)t$) is that while the former serves as the argument of a type $\langle 1 \rangle$ GQ, the latter serves as a function which requires a type $\langle 1 \rangle$ GQ as its argument.

3 Basic Formal Framework for Reciprocal Items

I propose that the internal reading of sentences containing the reciprocal items should also be treated using the GNP EACH OTHER. In this section, I will first propose a formal framework that makes explicit use of EACH OTHER. Then I will refine the framework by introducing new GNPs that make EACH OTHER implicit.

Consider (1) (and its equivalent form (2)) again. I propose to denote this sentence by the expression EACH OTHER$(R)(Q)$ where Q should be the type $\langle 1 \rangle$ GQ $\text{AND}_{3,(et)t}(I_{\mathbf{j}}, I_{\mathbf{p}}, I_{\mathbf{m}})$, while R should be a binary predicate denoting "read the same books as". Now the question is how to determine this binary predicate.

To answer the aforesaid question, we may draw reference from [1]'s study on structured quantifiers. According to [1], the structure "the same ... as" in the sentence "The same boys sang as danced" can be seen as a type $\langle 1, 1^2 \rangle$ GQ with the following denotation:

$$\text{THE SAME AS} = \lambda A_{et} \lambda (B_{et}, C_{et})[\text{SAME}(A' \cap B', A' \cap C')] \quad (16)$$

Note that THE SAME AS given above is an atomic logical term whose denotation contains another logical term SAME, with the following denotation (in what follows S and T are variables of sets):

$$\text{SAME} = \lambda(S, T)[S = T] \quad (17)$$

According to [18], while the structure "the same ... as" in the phrase "read the same books as" cannot be seen as a GQ, it is one of the "generalized comparative determiners" which constitute a subtype of GNPs (and will henceforth be treated as GNPs), and include, apart from "the same ... as", such structures as "different ... than", "older ... than" (where ... stands for a noun). This subtype of GNPs satisfies a property called "argument invariance for unary determiners" (D1AI), which can be seen as a defining property of this subtype of GNPs.

In this paper, I modify [18]'s theory a bit, and propose to denote "the same ... as" in "read the same books as" as a type $\langle 1, 2 : 2 \rangle$ GNP with the following denotation:

$$(\text{THE SAME AS})_{N,2} = \lambda P_{et} \lambda R_{e^2t} \lambda (x_e, y_e)[\text{SAME} \\ (P' \cap (I_x)\text{nom}(R)', P' \cap (I_y)\text{nom}(R)')] \quad (18)$$

where the subscripts N and 2 represent the fact that the internal reading is triggered by a plural noun phrase and “the same” appears in the 2nd argument (i.e. object) position of the binary predicate R, respectively. Note that the GNP defined above satisfies a modified version of the property D1AI defined as follows: a type $\langle 1, 2 : 2\rangle$ GNP F satisfies D1AI iff for any unary predicate P, binary predicate R and individuals a, b, c, d, if $P' \cap \{w : R(a, w) = 1\} = P' \cap \{w : R(b, w) = 1\}$ and $P' \cap \{w : R(c, w) = 1\} = P' \cap \{w : R(d, w) = 1\}$, then $F(P)(R)(a, c) = F(P)(R)(b, d)$.

Having defined the GNP above, we then apply it to the unary predicate **book** and the binary predicate **read** to obtain the denotation of “read the same books as” as follows:

$$(\textsc{the same as})_{N,2}(\mathbf{book})(\mathbf{read}) \tag{19}$$

Finally, we can determine the denotation of (1) as follows:

$$\textsc{each other}((\textsc{the same as})_{N,2}(\mathbf{book})(\mathbf{read}))(\textsc{and}_{3,(et)t}(I_{\mathbf{j}}, I_{\mathbf{p}}, I_{\mathbf{m}})) \tag{20}$$

It can be shown that the expression above is true if and only if the following is true:

$$\begin{aligned}&\{(\mathbf{j}, \mathbf{p}), (\mathbf{j}, \mathbf{m}), (\mathbf{p}, \mathbf{j}), (\mathbf{p}, \mathbf{m}), (\mathbf{m}, \mathbf{j}), (\mathbf{m}, \mathbf{p})\} \subseteq \\ &\{(x, y) : \mathbf{book}' \cap \{w : \mathbf{read}(x, w) = 1\} = \mathbf{book}' \cap \{w : \mathbf{read}(y, w) = 1\}\}\end{aligned} \tag{21}$$

The expression above means that any two (different) individuals among John, Peter and Mary are such that the books that one read are the same as the books that the other read, which is exactly what (1) means.

Note that the expression above treats “the same” as having an exhaustive meaning, i.e. (1) is true if and only if the books John read, the books Peter read and the books Mary read are exactly the same, no more and no less. This is in line with the treatments adopted by a number of scholars including [7], [9], [12], [17], [20], etc, but is different from [3], which adopts a non-exhaustive treatment under which (1) would be true if there are some books that John, Peter and Mary all read, and it is possible that any one of them might have read some other books that the other have not read. As pointed out by [12], [3]'s non-exhaustive treatment is incorrect. An evidence for the incorrectness is provided by the following contradictory sentence given in [20]:

∗ Leo and Lea read the same books and
Leo, but not Lea, read in addition *Exciting Humor*. (22)

If [3]'s non-exhaustive treatment were correct, then the first clause above would not rule out the possibility that Leo read some books that Lea did not read, and the sentence above would not be contradictory.

The internal reading of sentences containing the other reciprocal items can also be treated analogously. More specifically, I propose that all these items (each with a suitable preposition) should be treated as type $\langle 1, 2 : 2\rangle$ GNPs. For example, "separate ... from" is denoted by the following GNP:

$$(\textsc{separate from})_{N,2} = \lambda P_{et}\lambda R_{e^2t}\lambda(x_e, y_e)[\mathbf{separate}\ (P' \cap (I_x)\mathrm{nom}(R)', P' \cap (I_y)\mathrm{nom}(R)')] \quad (23)$$

A justification for this treatment is that $(\textsc{separate from})_{N,2}$ also satisfies the aforesaid modified version of D1AI. The only difference between (18) and (23) is that while SAME is a logical term with a denotation independent of models, **separate** is a non-logical term whose denotation is dependent on models, i.e. $\mathbf{separate}(S, T)$ is true in a particular model if and only if S and T are singletons and the unique members in these two singletons are separate entities in that model.

Based on the above, the denotation of sentence (4) given in Section 1 can be determined as follows:

$$\textsc{each other}((\textsc{separate from})_{N,2}(\mathbf{floor})(\mathbf{live\ on}))(\textsc{and}_{3,(et)t}(I_{\mathbf{j}}, I_{\mathbf{p}}, I_{\mathbf{m}})) \quad (24)$$

It can be shown that the expression above is true if and only if the following is true:

$$\{(\mathbf{j}, \mathbf{p}), (\mathbf{j}, \mathbf{m}), (\mathbf{p}, \mathbf{j}), (\mathbf{p}, \mathbf{m}), (\mathbf{m}, \mathbf{j}), (\mathbf{m}, \mathbf{p})\} \subseteq \{(x, y) : \mathbf{separate}\ (\mathbf{floor}' \cap \{w : \mathbf{live\ on}(x, w) = 1\}, \mathbf{floor}' \cap \{w : \mathbf{live\ on}(y, w) = 1\})\} \quad (25)$$

The expression above means that any two of John, Peter and Mary are such that the floor on which one lives is separate from the floor on which the other lives, which is exactly what (4) means.

The remaining reciprocal items can also be treated in an analogous way. Thus, in addition to $(\textsc{the same as})_{N,2}$ and $(\textsc{separate from})_{N,2}$, we also have $(\textsc{different than})_{N,2}$, $(\textsc{opposite to})_{N,2}$, $(\textsc{similar to})_{N,2}$, $(\textsc{adjacent to})_{N,2}$, $(\textsc{complementary to})_{N,2}$, $(\textsc{connected with})_{N,2}$, $(\textsc{disjoint from})_{N,2}$, etc, which can all be shown to satisfy the modified version of D1AI. In this way, we have greatly expanded the inventory of type $\langle 1, 2 : 2\rangle$ GNPs first studied in [18].

In the basic framework introduced above, I have been explicitly using the GNP EACH OTHER in the denotation of the sentences discussed above. However, the surface syntactic structure of (1), for instance, does not contain "each other". To further refine the framework to make it more in line with the surface syntactic structure, I propose to introduce a new GNP defined as follows:

$$(\textsc{internal the same})_{N,2}$$

$$= \lambda P_{et} \lambda R_{e^2t} \lambda Q_{(et)t}[\textsc{each other}((\textsc{the same as})_{N,2}(P)(R))(Q)]$$
$$= \lambda P_{et} \lambda R_{e^2t} \lambda Q_{(et)t}[\exists X \in \mathrm{Wit}(Q)[|X| \geq 2 \wedge X^2 - Id_X^2 \subseteq \{(x,y) : P' \cap \{w : R(x,w) = 1\} = P' \cap \{w : R(y,w) = 1\}\}]] \quad (26)$$

This new GNP is of type $\langle 1, 2 : \langle 1 \rangle \rangle$. By using this new GNP, we can now write down the denotation of (1) as follows:

$$(\textsc{internal the same})_{N,2}(\mathbf{book})(\mathbf{read})(\textsc{and}_{3,(et)t}(I_{\mathbf{j}}, I_{\mathbf{p}}, I_{\mathbf{m}})) \quad (27)$$

Note that this expression does not contain EACH OTHER explicitly, which is in line with the surface syntactic structure of (1). Moreover, one can show that by applying the denotation of this new GNP given in (26) to the expression above, we obtain (21), which is the correct truth condition of (1).

In an analogous way, we may also introduce other type $\langle 1, 2 : \langle 1 \rangle \rangle$ GNPs such as $(\textsc{internal separate})_{N,2}$, $(\textsc{internal different})_{N,2}$, etc. In this way, we can denote sentences with internal reading by using GNPs carrying a reciprocal meaning without using the GNP EACH OTHER explicitly. For example, the denotation of (4) can now be written as

$$(\textsc{internal separate})_{N,2}(\mathbf{floor})(\mathbf{live\ on})(\textsc{and}_{3,(et)t}(I_{\mathbf{j}}, I_{\mathbf{p}}, I_{\mathbf{m}})) \quad (28)$$

4 Further Refinements of the Framework

In this section, I will further refine the basic framework formulated in the previous section to handle several cases that are different from those discussed above in one way or another.

4.1 Reciprocal Items in non-Object Positions

The first case concerns reciprocal items appearing in non-object positions such as the following:

The same universities admitted John, Peter and Mary. (29)

in which "the same" appears in the 1st argument (i.e. subject) position of the verb "admitted". To handle this sentence, we can change the GNP $(\textsc{the same as})_{N,2}$ given in (18) to $(\textsc{the same as})_{N,1}$ where the subscript 1 represents the fact that "the same" appears in the 1st argument position. To define $(\textsc{the same as})_{N,1}$, we only need to change $(I_x)_{\mathrm{nom}}$ and $(I_y)_{\mathrm{nom}}$ in (18) to $(I_x)_{\mathrm{acc}}$ and $(I_y)_{\mathrm{acc}}$, respectively. Moreover, we also need to change $(\textsc{internal the same})_{N,2}$ given in (26) to $(\textsc{internal the same})_{N,1}$, with the following definition:

$$(\textsc{internal the same})_{N,1}$$
$$= \lambda P_{et} \lambda R_{e^2t} \lambda Q_{(et)t}[\textsc{each other}((\textsc{the same as})_{N,1}(P)(R))(Q)]$$
$$= \lambda P_{et} \lambda R_{e^2t} \lambda Q_{(et)t}[\exists X \in \mathrm{Wit}(Q)[|X| \geq 2 \wedge X^2 - Id_X^2 \subseteq \{(x,y) : P' \cap \{w : R(w,x) = 1\} = P' \cap \{w : R(w,y) = 1\}\}]] \quad (30)$$

With this GNP, we can write down the denotation of (29) as follows:

$$(\textsc{internal the same})_{N,1}(\mathbf{university})(\mathbf{admit})(\textsc{and}_{3,(et)t}(I_{\mathbf{j}}, I_{\mathbf{p}}, I_{\mathbf{m}})) \quad (31)$$

It can be shown that the expression above is true if and only if the following is true:

$$\begin{aligned} &\{(\mathbf{j},\mathbf{p}),(\mathbf{j},\mathbf{m}),(\mathbf{p},\mathbf{j}),(\mathbf{p},\mathbf{m}),(\mathbf{m},\mathbf{j}),(\mathbf{m},\mathbf{p})\} \\ \subseteq &\{(x,y) : \mathbf{university}' \cap \{w : \mathbf{admit}(w,x) = 1\} \\ &= \mathbf{university}' \cap \{w : \mathbf{admit}(w,y) = 1\}\} \end{aligned} \quad (32)$$

The expression above is true if and only if any two of John, Peter and Mary are such that the universities that admitted one are the same as the universities that admitted the other, which is exactly what (29) means. Sentences in which the reciprocal items appear in other syntactic positions, such as the indirect object position of a ternary predicate, can be treated in an analogous manner.

4.2 Conjoined Phrases containing Reciprocal Items

The second case concerns conjoined phrases containing reciprocal items such as the following:

John, Peter and Mary borrowed the same books and bought different magazines. (33)

with two phrases "borrowed the same books" and "bought different magazines" each containing a different reciprocal item conjoined. According to [1], there is a weak and a strong versions of "different" with the following denotations (where the subscript "w" and "s" stand for "weak" and "strong", respectively), i.e.

$$\textsc{different}_w = \lambda(S,T)[S \neq T] \quad (34)$$

$$\textsc{different}_s = \lambda(S,T)[S \cap T = \emptyset] \quad (35)$$

One may choose whichever version that fits the meaning of a particular sentence. In what follows, I arbitrarily choose the strong version. To handle (33), we have to use an appropriate version of the polymorphic $\textsc{and}_{n,\tau}$ to conjoin the appropriate expressions. In this case, we can introduce the following GNP:

$$\begin{aligned} &(\textsc{internal the same and different}_s)_{N,2} \\ = &\lambda((P_1)_{et},(P_2)_{et})\lambda((R_1)_{e^2t},(R_2)_{e^2t})\lambda Q_{(et)t}[\textsc{each other} \\ &(\textsc{and}_{2,e^2t}((\textsc{the same as})_{N,2}(P_1)(R_1),(\textsc{different}_s \textsc{ than})_{N,2}(P_2)(R_2)))(Q)] \\ = &\lambda((P_1)_{et},(P_2)_{et})\lambda((R_1)_{e^2t},(R_2)_{e^2t})\lambda Q_{(et)t}[\exists X \in \mathrm{Wit}(Q)[|X| \geq 2 \\ &\wedge X^2 - Id_X^2 \subseteq \{(x,y) : P_1' \cap \{w : R_1(x,w) = 1\} = P_1' \cap \{w : R_1(y,w) = 1\} \\ &\wedge P_2' \cap \{w : R_2(x,w) = 1\} \cap \{w : R_2(y,w) = 1\} = \emptyset\}]] \end{aligned} \quad (36)$$

The above is a type $\langle 1^2, 2^2 : \langle 1\rangle\rangle$ GNP because it requires an orderer pair of unary predicates P_1, P_2 and an orderer pair of binary predicates R_1, R_2 as inputs and outputs a characteristic function of type $\langle 1\rangle$ GQs. Note that in the expression above, I use $\textsc{and}_{2,e^2t}$ to conjoin two binary predicates $(\textsc{the same as})_{N,2}(P_1)(R_1)$ and $(\textsc{different}_s\ \textsc{than})_{N,2}(P_2)(R_2)$[3]. This reflects the fact that what the conjunction "and" conjoins in (33) are the two verb phrases "borrowed the same books" and "bought different magazines", not just the reciprocal items "the same" and "different". With the GNP above, we can write down the denotation of (33) as follows:

$$(\textsc{internal the same and different}_s)_{N,2}(\mathbf{book}, \mathbf{magazine})$$
$$(\mathbf{borrow}, \mathbf{buy})(\textsc{and}_{3,(et)t}(I_{\mathbf{j}}, I_{\mathbf{p}}, I_{\mathbf{m}})) \quad (37)$$

It can be shown that the expression above is true if and only if the following is true:

$$\{(\mathbf{j}, \mathbf{p}), (\mathbf{j}, \mathbf{m}), (\mathbf{p}, \mathbf{j}), (\mathbf{p}, \mathbf{m}), (\mathbf{m}, \mathbf{j}), (\mathbf{m}, \mathbf{p})\} \subseteq \{(x, y) :$$
$$\mathbf{book'} \cap \{w : \mathbf{borrow}(x, w) = 1\} = \mathbf{book'} \cap \{w : \mathbf{borrow}(y, w) = 1\}$$
$$\wedge\ \mathbf{magazine'} \cap \{w : \mathbf{buy}(x, w) = 1\} \cap \{w : \mathbf{buy}(y, w) = 1\} = \emptyset\} \quad (38)$$

The expression above means that any two of John, Peter and Mary are such that the books that one borrowed are the same as the books that the other borrowed, and the magazines that one bought are completely different from the magazines that the other bought, which is exactly what (33) means under a strong sense of "different". Sentences with conjoined phrases containing other reciprocal items can be treated in an analogous manner.

4.3 Downward Monotonic and Non-monotonic GQs

The third case concerns downward monotonic and non-monotonic type $\langle 1\rangle$ GQs, such as the following:

Fewer than 3 persons read the same books. (39)

The denotation of EACH OTHER introduced in (13) is only applicable to cases where Q is upward monotonic such as $\textsc{and}_{3,(et)t}(I_{\mathbf{j}}, I_{\mathbf{p}}, I_{\mathbf{m}})$, AT LEAST 3 (**person**), etc. When Q is downward monotonic or non-monotonic, (13) is not adequate. For the definitions of upward monotonic, downward monotonic and non-monotonic GQs, please refer to the standard GQT literature such as [10]. To obtain the denotation applicable to all type $\langle 1\rangle$ GQs, I propose to borrow ideas from [2] and modify (13) as follows:

[3] Under the e-t notation, a binary predicate is of type e^2t.

$$\text{EACH OTHER}^{\#} = \lambda R_{e^2t}\lambda Q_{(et)t}\ \ [(\exists X \in \text{Wit}(Q)[|X| \geq 2 \wedge X^2 - Id^2_X \subseteq R'] \\ \vee\ \text{Top}(Q) \cap S = \emptyset)\ \wedge\ S \in Q'] \tag{40}$$

where S is the following set denoting "individuals that enter into a mutual R relation":

$$S = \{z : \exists X[z \in X \wedge |X| \geq 2 \wedge X^2 - Id^2_X \subseteq R']\} \tag{41}$$

In (40), the first conjunct ($\exists X \cdots = \emptyset$) corresponds to the "witness operator" discussed in [2]. Within this conjunct, the disjunct $\exists X \cdots \subseteq R']$ is to specify the witness set whereas the disjunct $\text{Top}(Q) \cap S = \emptyset$ is to handle cases that do not require a witness set. The second conjunct $S \in Q'$ corresponds to the "counting operator" discussed in [2] and is used to count the number of individuals that enter into a mutual R relation.

The relation between (13) and (40) is given in the following proposition.

Proposition 1. *Let Q be a non-tautological upward monotonic type $\langle 1 \rangle$ GQ. Then for any binary predicate R,* EACH OTHER$(R)(Q) = 1$ *iff* EACH OTHER$^{\#}(R)(Q)$ $= 1$.

Proof. Let EACH OTHER$(R)(Q) = 1$, which is equivalent to $\exists X \in \text{Wit}(Q)[|X| \geq 2 \wedge X^2 - Id^2_X \subseteq R']$. From this we can easily deduce the disjunction $\exists X \in \text{Wit}(Q)[|X| \geq 2 \wedge X^2 - Id^2_X \subseteq R'] \vee \text{Top}(Q) \cap S = \emptyset$. Next I show that this set X satisfies $X \subseteq S$. Let $z \in X$, then it is obvious that there is an X such that $z \in X \wedge |X| \geq 2 \wedge X^2 - Id^2_X \subseteq R'$, and so according to the definition of S, we have $z \in S$. It thus follows that $X \subseteq S$. Now from the fact that $X \in \text{Wit}(Q)$ and the definition of witness sets, we have $X \in Q'$. But then by the upward monotonicity of Q, we also have $S \in Q'$. Thus, I have shown that $(\exists X \in \text{Wit}(Q)[|X| \geq 2 \wedge X^2 - Id^2_X \subseteq R'] \vee \text{Top}(Q) \cap S = \emptyset) \wedge S \in Q'$, which is equivalent to EACH OTHER$^{\#}(R)(Q) = 1$.

Let EACH OTHER$^{\#}(R)(Q) = 1$. Then we have $S \in Q'$ and the disjunction $\exists X \in \text{Wit}(Q)[|X| \geq 2 \wedge X^2 - Id^2_X \subseteq R'] \vee \text{Top}(Q) \cap S = \emptyset$ both true. Suppose by way of contradiction, $\text{Top}(Q) \cap S = \emptyset$. According to the definition of topic sets, we have $\text{Top}(Q) \in \text{Li}(Q)$. So from $S \in Q'$ and the definition of live-on sets, we can deduce $S \cap \text{Top}(Q) \in Q'$, which is equivalent to $\emptyset \in Q'$. But then by the upward monotonicity of Q, we have $X \in Q'$ for any subset X of the domain of discourse, and so Q is tautological, contrary to assumption. It thus follows that $\text{Top}(Q) \cap S \neq \emptyset$, and so the disjunction $\exists X \in \text{Wit}(Q)[|X| \geq 2 \wedge X^2 - Id^2_X \subseteq R'] \vee \text{Top}(Q) \cap S = \emptyset$ reduces to $\exists X \in \text{Wit}(Q)[|X| \geq 2 \wedge X^2 - Id^2_X \subseteq R']$, which is equivalent to EACH OTHER$(R)(Q) = 1$. □

The proposition above shows that when Q is non-tautological upward monotonic, then (40) reduces to (13) and so it is more convenient and justifiable to use the simpler expression given in (13) to denote "each other" when Q is non-tautological upward monotonic, just as we did above.

We can now write down the denotation of (39) as follows:

$$(\textsc{internal the same})_{N,2}(\mathbf{book})(\textsc{fewer than } 3(\mathbf{person})) \quad (42)$$

where the definition of $(\textsc{internal the same})_{N,2}$ given in (26) has to be modified as follows:

$$\begin{aligned}&(\textsc{internal the same})_{N,2}\\ &= \lambda P_{et}\lambda R_{e^2t}\lambda Q_{(et)t}[\textsc{each other}^{\#}((\textsc{the same as})_{N,2}(P)(R))(Q)]\end{aligned} \quad (43)$$

It can be shown that (42) is true if and only if the following is true[4]:

$$\begin{aligned}&(\exists X \in \mathrm{Wit}(\textsc{fewer than } 3(\mathbf{person}))[|X| \geq 2 \wedge X^2 - Id_X^2 \subseteq\\ &\{(x,y) : \mathbf{book'} \cap \{w : \mathbf{read}(x,w) = 1\} = \mathbf{book'} \cap \{w : \mathbf{read}(y,w) = 1\}\}]\\ &\vee \mathbf{person'} \cap S = \emptyset) \ \wedge \ |\mathbf{person'} \cap S| < 3\end{aligned} \quad (44)$$

where S is the following set denoting "individuals who read the same books":

$$\begin{aligned}S = \{z : \exists X[z \in X \wedge |X| \geq 2 \wedge X^2 - Id_X^2 \subseteq \{(x,y) :\\ \mathbf{book'} \cap \{w : \mathbf{read}(x,w) = 1\} = \mathbf{book'} \cap \{w : \mathbf{read}(y,w) = 1\}\}]\}\end{aligned} \quad (45)$$

The first conjunct of (44), namely $(\exists X \cdots = \emptyset)$, consists of two disjuncts because (39) can be true in two possible ways and are thus represented by two different disjuncts. The first way is that there is a specific group (i.e. a witness set) consisting of fewer than 3 persons reading the same books, and this is represented by the disjunct $\exists X \cdots = 1\}\}]$. The second way is that no person read the same books, and this is represented by the disjunct $\mathbf{person'} \cap S = \emptyset$. The second conjunct of (44), namely $|\mathbf{person'} \cap S| < 3$, counts the number of people reading the same books. Thus, (44) gives us the correct truth condition of (39). Sentences with other downward monotonic or non-monotonic type $\langle 1 \rangle$ GQs can be treated in an analogous manner.

4.4 Weaker Reciprocal Meaning

The fourth case concerns sentences with a weaker reciprocal meaning such as the following:

John, Peter and Mary live in adjacent buildings. (46)

Using the framework developed so far, one may think that we can denote this sentence using a type $\langle 1, 2 : \langle 1 \rangle\rangle$ GNP $(\textsc{internal adjacent})_{N,2}$ with definition similar to (43) that contains the GNP $\textsc{each other}^{\#}$. However, the use of

[4] It can be shown that $\mathrm{Top}(\textsc{fewer than } 3(\mathbf{person})) = \mathbf{person'}$.

EACH OTHER$^{\#}$ is too strong for the sentence (46). Note that three buildings normally cannot be adjacent to each other in the strongest sense of "each other" given in (40) (or (13)) above because if building A is adjacent to building B and building B is adjacent to building C, then building A cannot be adjacent to building C (unless they are arranged in a circle).

To handle the sentence above, we first define a weaker version of "each other" which is notated EACH OTHER^{+}. This GNP differs from EACH OTHER$^{\#}$ in that the binary predicate R' in (40) and (41) is changed to $(R' \cap X^2 - Id_X^2)^+$, which represents the transitive closure of $R' \cap X^2 - Id_X^2$, i.e. the smallest transitive relation containing $R' \cap X^2 - Id_X^2$.

We then define a type $\langle 1, 2 : \langle 1 \rangle \rangle$ GNP $(\text{INTERNAL ADJACENT})_{N,2}$ as follows:

$$\begin{aligned}&(\text{INTERNAL ADJACENT})_{N,2}\\ &= \lambda P_{et} \lambda R_{e^2t} \lambda Q_{(et)t} [\text{EACH OTHER}^{+}((\text{ADJACENT TO})_{N,2}(P)(R))(Q)]\end{aligned} \tag{47}$$

where $(\text{ADJACENT TO})_{N,2}$ is a type $\langle 1, 2 : 2 \rangle$ GNP whose definition is similar to (23) but contains the non-logical term **adjacent** instead of **separate**. By using this GNP, we can write down the denotation of (46) as the following expression:

$$(\text{INTERNAL ADJACENT})_{N,2}(\mathbf{building})(\mathbf{live\ in})(\text{AND}_{3,(et)t}(I_{\mathbf{j}}, I_{\mathbf{p}}, I_{\mathbf{m}})) \tag{48}$$

It can be shown that the expression above is true if and only if the following is true:

$$\begin{aligned}&\{(\mathbf{j},\mathbf{p}),(\mathbf{j},\mathbf{m}),(\mathbf{p},\mathbf{j}),(\mathbf{p},\mathbf{m}),(\mathbf{m},\mathbf{j}),(\mathbf{m},\mathbf{p})\} \subseteq (\{(x,y) : \mathbf{adjacent}\\ &(\mathbf{building}' \cap \{w : \mathbf{live\ in}(x,w)=1\}, \mathbf{building}' \cap \{w : \mathbf{live\ in}(y,w)=1\})\}\\ &\cap\ \{(\mathbf{j},\mathbf{p}),(\mathbf{j},\mathbf{m}),(\mathbf{p},\mathbf{j}),(\mathbf{p},\mathbf{m}),(\mathbf{m},\mathbf{j}),(\mathbf{m},\mathbf{p})\})^+\end{aligned} \tag{49}$$

In the expression above, $\{(x,y) : \cdots (\mathbf{m},\mathbf{p})\}$ represents the adjacent relation (if any) between the buildings in which John, Peter and Mary live, $(\{(x,y) : \cdots (\mathbf{m},\mathbf{p})\})^+$ represents the transitive closure of this relation, which is also a relation. Now if John's building is adjacent to Peter's building, which is in turn adjacent to Mary's building, then John's building and Mary's building do not in general stand in the adjacent relation. But they do stand in the aforesaid transitive closure relation. Thus, the expression above gives us the correct truth condition of (46) and this shows that EACH OTHER^{+} gives the correct reciprocal relation associated with "adjacent".

Apart from EACH OTHER^{+}, there are other reciprocal relations studied in [5] and [11] that are also weaker than EACH OTHER$^{\#}$. Like "adjacent", other reciprocal items may be associated with some of these weaker reciprocal relations. A thorough understanding of the specific reciprocal relation denoted by each reciprocal item is beyond the scope of this paper and is left for future research.

4.5 Non-NP Triggers

So far in this paper I have only considered internal reading triggered by plural NPs (including conjoined NPs and plural quantified NPs). However, as pointed out by a number of scholars including [3], [4], [12], [17], [20] etc, the internal reading can also be triggered by other plural syntactic categories, as exemplified by the following sentences:

$$\text{The same boys scolded, chased and beat John.} \tag{50}$$

$$\text{John ate different food in the morning, afternoon and evening.} \tag{51}$$

In (50), the trigger is the conjoined verbs "scolded, chased and beat". To handle this case, we can make use of the notion of raised VP briefly introduced in Section 2. According to [22], from each VP of type et, one may derive a raised VP of type $((et)t)t$. For (50), we first write down the denotation of the VP "scolded John" as $(I_{\mathbf{j}})_{\text{acc}}(\mathbf{scold})$. For convenience, hereinafter I use **sj** (representing "scolded John") as the short form of this unary predicate. From **sj** of type et, we can derive a raised VP of type $((et)t)t$ with the following denotation[5]:

$$I_{\mathbf{sj}} = \lambda Q_{(et)t}[Q(\mathbf{sj}) = 1] \tag{52}$$

Similar definitions can also be given to $I_{\mathbf{cj}}$ (for "chased John") and $I_{\mathbf{bj}}$ (for "beat John"). Note that the definition above is very similar to that of the Montagovian individual $I_{\mathbf{j}}$ given in (5), and can be seen as a "Montagovian VP". In this way, we are treating et as a basic type. Moreover, certain notions and functions associated with type $\langle 1 \rangle$ GQs can be extended to these Montagovian VPs.

First, the notions of live-on sets, topic sets and witness sets can all be extended to the Montagovian VPs. For example, by following similar steps for proving (12) given in Section 2, one can easily prove that

$$\text{Wit}(\textsc{and}_{3,((et)t)t}(I_{\mathbf{sj}}, I_{\mathbf{cj}}, I_{\mathbf{bj}})) = \{\{\mathbf{sj}, \mathbf{cj}, \mathbf{bj}\}\} \tag{53}$$

Second, the formal framework developed above for handling internal reading triggered by plural NPs can be extended to handle (50). But first we have to modify the GNP $(\textsc{the same as})_{N,2}$ given in (18) to obtain the following GNP:

$$(\textsc{the same as})_{V,1} = \lambda P_{et}\lambda(F_{et}, G_{et})[\textsc{same}(P' \cap F', P' \cap G')] \tag{54}$$

where the subscript V represents the fact that the internal reading is triggered by a plural verb phrase.

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[5] According to [22], the raised VP derived from **sj** is notated $\mathbf{sj}^R$. In this paper, I use the notation $I_{\mathbf{sj}}$ instead to highlight the similarity between Montagovian individuals and raised VPs derived from ordinary VPs.

Furthermore, we also need to modify the GNP EACH OTHER$^{\#}$, because EACH OTHER$^{\#}$ given in (40) is of type $(e^2t)((et)t)t$, whereas we now need a function of type $((et)^2t)(((et)t)t)t$. But apart from this type change, the meaning of "each other" in fact has not changed. A solution to this problem is to change EACH OTHER$^{\#}$ to the following polymorphic function:

$$\text{EACH OTHER}^{\#} = \lambda R_{\tau^2 t}\lambda Q_{(\tau t)t}[\ldots] \tag{55}$$

where ... is the same as its corresponding part in (40). The function given above is of type $(\tau^2t)((\tau t)t)t$, where τ is a generic type. By instantiating τ as e, we get our good old friend (40). By instantiating τ as et, we get EACH OTHER$^{\#}$ with the desired type for handling (50). Again, to make EACH OTHER$^{\#}$ implicit, we can define the following GNP:

$$\begin{aligned}&(\text{INTERNAL THE SAME})_{V,1}\\ &= \lambda P_{et}\lambda V_{((et)t)t}[\text{EACH OTHER}^{\#}((\text{THE SAME AS})_{V,1}(P))(V)]\end{aligned} \tag{56}$$

The denotation of (50) can now be written as follows[6]:

$$(\text{INTERNAL THE SAME})_{V,1}(\mathbf{boy})(\text{AND}_{3,((et)t)t}(I_{\mathbf{sj}}, I_{\mathbf{cj}}, I_{\mathbf{bj}})) \tag{57}$$

It can be shown that this expression is true if and only if the following is true:

$$\begin{aligned}&\{(\mathbf{sj},\mathbf{cj}),(\mathbf{sj},\mathbf{bj}),(\mathbf{cj},\mathbf{sj}),(\mathbf{cj},\mathbf{bj}),(\mathbf{bj},\mathbf{sj}),(\mathbf{bj},\mathbf{cj})\} \subseteq\\ &\{(F,G) : \mathbf{boy}' \cap F' = \mathbf{boy}' \cap G'\}\end{aligned} \tag{58}$$

The expression above means that any two actions among scolding John, chasing John and beating John are such that the boys engaged in one action are the same as the boys engaged in the other action, which is exactly what (50) means.

In (51), the trigger is the conjoined temporal adverbials "in the morning, afternoon and evening". To handle this case without making great changes to the existing framework, we can draw reference from [6] and introduce a new type i for time intervals and treat "morning", "afternoon" and "evening" as individual items of this new basic type (here I ignore the definite article "the"). We can then derive a "Montagovian interval" of type $(it)t$. For example, from **mo** (short for "morning") of type i, we have $I_{\mathbf{mo}}$ of type $(it)t$ with the following denotation:

[6] If one thinks that (57) is not in line with the surface syntactic structure of (50) by having 3 occurrences of **j**, one can use λ-abstraction to abstract out $I_{\mathbf{j}}$ to obtain a function and then reapply the function back to $I_{\mathbf{j}}$. In this way, we can replace $\text{AND}_{3,((et)t)t}(I_{\mathbf{sj}}, I_{\mathbf{cj}}, I_{\mathbf{bj}})$ in (57) by $\lambda Q_{(et)t}[\text{AND}_{3,((et)t)t}(Q\text{acc}(\mathbf{scold})^R, Q\text{acc}(\mathbf{chase})^R, Q\text{acc}(\mathbf{beat})^R)](I_j)$, which contains only 1 occurrence of I_j.

$$I_{\mathbf{mo}} = \lambda P_{it}[P(\mathbf{mo}) = 1] \tag{59}$$

Similar definitions can also be given to $I_{\mathbf{af}}$ (for "afternoon") and $I_{\mathbf{ev}}$ (for "evening"). Moreover, the notions of live-on sets, topic sets and witness sets can all be extended to the Montagovian intervals. For example, it can be shown that

$$\text{Wit}(\text{AND}_{3,(it)t}(I_{\mathbf{mo}}, I_{\mathbf{af}}, I_{\mathbf{ev}})) = \{\{\mathbf{mo}, \mathbf{af}, \mathbf{ev}\}\} \tag{60}$$

We also need to define the following function of type itt denoting the temporal preposition "in":

$$\text{IN} = \lambda t_i \lambda p_t[p \text{ holds in interval } t] \tag{61}$$

To handle (51), we define the following GNP:

$$\begin{aligned}&(\text{DIFFERENT}_s \text{ THAN})_{T,2}\\ &= \lambda P_{et}\lambda Q_{(et)t}\lambda R_{e^2t}\lambda O_{itt}\lambda(s_i, t_i)[\text{DIFFERENT}_s(P' \cap \{w : O(s, \{z : R(z, w) = 1\} \in Q') = 1\},\\ &P' \cap \{w : O(t, \{z : R(z, w) = 1\} \in Q') = 1\})]\end{aligned} \tag{62}$$

where the subscript T represents the fact that the internal reading is triggered by a plural temporal adverbial. Furthermore, the GNP EACH OTHER$^{\#}$ has to be of type $(i^2t)((it)t)t$. By instantiating the generic type τ as i in (55), we easily get EACH OTHER$^{\#}$ with the desired type. We next define the following GNP:

$$\begin{aligned}&(\text{INTERNAL DIFFERENT}_s)_{T,2} = \lambda P_{et}\lambda Q_{(et)t}\lambda R_{e^2t}\lambda O_{itt}\lambda T_{(it)t}\\ &[\text{EACH OTHER}^{\#}((\text{DIFFERENT}_s \text{ THAN})_{T,2}(P)(Q)(R)(O))(T)]\end{aligned} \tag{63}$$

The denotation of (51) can now be written as follows:

$$(\text{INTERNAL DIFFERENT}_s)_{T,2}(\mathbf{food})(I_{\mathbf{j}})(\mathbf{eat})(\text{IN})(\text{AND}_{3,(it)t}(I_{\mathbf{mo}}, I_{\mathbf{af}}, I_{\mathbf{ev}})) \tag{64}$$

It can be shown that this expression is true if and only if the following is true:

$$\begin{aligned}&\{(\mathbf{mo}, \mathbf{af}), (\mathbf{mo}, \mathbf{ev}), (\mathbf{af}, \mathbf{mo}), (\mathbf{af}, \mathbf{ev}), (\mathbf{ev}, \mathbf{mo}), (\mathbf{ev}, \mathbf{af})\} \subseteq \{(s, t) :\\ &\mathbf{food}' \cap \{w : \text{IN}(s, \mathbf{eat}(\mathbf{j}, w) = 1)\} \cap \{w : \text{IN}(t, \mathbf{eat}(\mathbf{j}, w) = 1)\} = \emptyset\}\end{aligned} \tag{65}$$

The expression above means that any two intervals among the morning, afternoon and evening are such that the food John ate in one interval is different from the food John ate in the other interval, which is exactly what (51) means. Sentences with other non-NP triggers can be treated in an analogous way provided that we introduce the appropriate basic types (like i in the previous case) and appropriate functions (like IN in the previous case).

5 Conclusion

In this paper, I have identified a number of reciprocal items and showed that the internal reading of sentences containing these items carries a reciprocal meaning. In view of this, I propose to denote such sentences by using the GNP EACH OTHER as well as a number of type $\langle 1, 2 : 2\rangle$ GNPs corresponding to the reciprocal items such as (THE SAME AS)$_{N,2}$. To refine the formal framework, I then define type $\langle 1, 2 : \langle 1\rangle\rangle$ GNPs such as (INTERNAL THE SAME)$_{N,2}$ to make EACH OTHER implicit. In this way, sentences with internal reading can be denoted by expressions that are in line with the surface syntactic structures of the sentences. I then discuss how to further refine the framework by considering reciprocal items appearing in non-object positions, conjoined phrases containing reciprocal items, downward monotonic and non-monotonic quantifiers, weaker reciprocal meaning and non-NP triggers.

Apart from providing a uniform formal framework, this paper has also enriched the theory relating to GNPs by greatly expanding the inventory of type $\langle 1, 2 : 2\rangle$ GNPs and introducing a new class of $\langle 1, 2 : \langle 1\rangle\rangle$ GNPs (as well as GNPs involving basic types other than e). Moreover, by considering non-NP triggers, this paper has also enriched the notion of polymorphism by treating EACH OTHER$^{\#}$ as a polymorphic function.

Before closing, I have to point out certain syntactic issues that this paper has not addressed[7]. The reciprocal items and "each other" are subject to different syntactic constraints in at least two respects. First, while "the same" (and other reciprocal items) can be triggered by a distributive plural NP, "each other" cannot, as illustrated by the following sentences:

Both boys recited the same poems. (66)

*Both boys hit each other. (67)

Second, while the reciprocal items can appear in subject position (as illustrated by (29)), "each other" cannot, as illustrated by the following sentence:

*Each other hit the boys. (68)

Sentence (67) shows that "each other" is associated with collective (as opposed to distributive) plural NPs. Thus, to treat "each other" properly, we have to introduce notions associated with plurality/collective quantification such as those given in [2], [14] and [16], to name just a few, and modify the denotation of EACH OTHER$^{\#}$ (or EACH OTHER^{+}) given above accordingly.

But then how can we explain the fact that "the same" can be associated with a distributive NP in (66) if we assume that the denotation of the GNP (INTERNAL THE SAME)$_{N,2}$ contains EACH OTHER$^{\#}$? A possible way is to add a

[7] My sincere thanks to an anonymous reviewer for pointing out the issues and inspiring the possible solutions.

distributivity operator into the denotation of (INTERNAL THE SAME)$_{N,2}$[8]. The syntactic difference between (66) and (67) can then be attributed to the presence/absence of this distributivity operator.

Similarly, the syntactic difference between (29) and (68) can be treated by adding certain syntactic/semantic elements into the denotations of EACH OTHER$^{\#}$ and/or (INTERNAL THE SAME)$_{N,1}$. The aforesaid syntactic difference can then be attributed to the presence/absence of these elements. As a thorough discussion of the treatment outlined above will involve introduction of new entities, notions and theories on top of those introduced in this paper, it will have to be left for future research.

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[8] Note that this distributivity operator is not an *ad hoc* device since it is also used in the formal treatment of plurality/collective quantification as introduced in the relevant literature.

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